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The effects of capillary forces on the island size distribution in two-fluid immiscible displacement flow in porous media

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Abstract. A stochastic algorithm is used to simulate the immiscible displacement of a wetting fluid by a non-wetting fluid in a porous medium represented by a two-dimensional network of interconnected capillaries. As the interface advances trapping of the displaced fluid occurs thereby creating isolated islands of the displaced fluid. The number of islands of size s is found to scale approximately as $s^{-\alpha}$, where α depends on the capillary number and the viscosity ratio. The effects of capillary forces on the island size distribution are also studied.

1. Introduction

Fingering is a phenomenon which occurs during the secondary and tertiary recovery of oil from underground reservoirs when water (or aqueous solutions containing surfactants, polymers, and/or alkali) is injected to displace the oil. The displacing phase tends to finger inside the displaced phase due to unfavourable viscosity ratios and due to heterogeneities in the reservoir. In some cases the fingers grow all over and intersect forming loops thereby leaving large regions of oil surrounded by water. These regions, called islands [1] or ganglia [2, 3], account for the so-called residual oil saturation.

In this work, a model which has been developed to simulate the immiscible displacement of a wetting fluid by a non-wetting one [4-6] is briefly described. The porous medium is represented by a two-dimensional square network of interconnected capillaries. The formation of islands of the displaced fluid and their size distribution are examined for the transition from viscous fingering to capillary fingering at low viscosity ratios and from stable displacement to capillary fingering at high viscosity ratios. The island size distribution is studied with respect to the capillary number, Ca, in order to examine the effects of capillary forces on the size distribution of the islands. The capillary number is defined as the ratio of viscous forces to capillary forces ($Ca = V\mu_{NW}/\gamma\cos\theta$), where V denotes the mean displacement velocity, μ_{NW} the viscosity of the non-wetting (displacing) fluid, γ the interfacial tension, and θ the contact angle. The viscosity ratio, M, is defined as the ratio of the displacing (non-wetting) fluid viscosity, μ_{NW}, to the displaced (wetting) fluid viscosity, μ_W ($M = \mu_{NW}/\mu_W$).

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2. The simulation algorithm

The present model is based on the three basic statistical models for two-fluid immiscible displacement flow in porous media, namely DLA (diffusion-limited aggregation), anti-DLA [7] and invasion percolation [8, 9], as well as on the notion of the phase diagram for two-fluid immiscible displacement flow in porous media [10, 11].

The DLA model describes the displacement of a viscous fluid by an almost inviscid fluid at a high capillary number. According to this model, random walkers are released in the displaced fluid, far away from the interface between the displacing and displaced fluids, and are allowed to wander on a lattice representing the porous medium with equal transition probabilities at each step. The motion of a random walker satisfies the Laplace equation for the pressure (i.e. $\nabla^2 P = 0$) in the porous medium [7]. When a random walker reaches an interfacial pore, the interface advances and the displacing fluid invades this pore.

The anti-DLA model describes the displacement of a viscous fluid by a more viscous fluid at high capillary numbers. According to this model, random walkers are released from within the displacing phase and are allowed to wander only in the region occupied by the displacing fluid. When a random walker comes into contact with the interface, the interface advances and the displacing fluid invades an interfacial pore. Therefore, both the DLA and anti-DLA models satisfy the Laplace equation for the pressure in the displaced and displacing phases, respectively. The absence of random walkers from one phase implies negligible viscous pressure gradients in that phase. The DLA and anti-DLA models represent two limiting cases of the displacement of one fluid by another at high capillary numbers.

The invasion percolation model describes the displacement of a fluid by another immiscible fluid at very low capillary numbers. Under these conditions, only capillary forces are significant and the interface moves along the paths of least resistance (drainage) or along the paths of largest driving force (imbibition). The driving force is represented by the capillary pressure, P_C, defined by

$$P_C = P_{NW} - P_W = 2\gamma \cos\theta / r \tag{1}$$

where P_{NW} and P_W denote the pressures at the interface within the non-wetting and wetting fluids respectively, and r is the radius of curvature of the (two-dimensional) interface. In this case the advancement of the interface is described by the invasion percolation model.

The ranges of validity of the above stochastic models have been mapped by Lenormand on a phase diagram having axes representing the viscosity ratio and the capillary number [10, 11]. Each of the above three models describes one of the three domains of the phase diagram. The boundaries of each domain are expressed by a limiting capillary number which is a function of the viscosity ratio and of certain physical properties of the network.

Considering the flow conservation equation at each pore (node) in the network for incompressible flow, namely

$$\sum_{i=1}^{4} Q_i = 0 \tag{2}$$

and assuming Poiseuille flow in each channel, the flow conservation equation for each phase may be expressed by

$$\sum_{i=1}^{4} g_i(P-P_i)=0 \qquad (3)$$

where g_i denotes the hydraulic conductivity of each channel, P the pressure at a central pore and P_i the pressure at a neighbouring pore. The above equation is satisfied also by a random walker which wanders on a square lattice from one pore to a neighbouring pore with a transition probability, p, which is given by

$$p=g_i\left(\sum_{i=1}^{4} g_i\right)^{-1}. \qquad (4)$$

Equation (4) results from equation (3) by solving for P and denoting by P and P_i the probabilities that the random walker will be on the central pore and a neighbouring pore i, respectively. When random walkers are allowed to wander in the displaced phase and to stick upon contact with the interface, this becomes a form of the original DLA model [7]. In contrast, when random walkers wander in the displacing phase this becomes a form of the anti-DLA model. In homogeneous networks the transition probabilities of the random walk are all equal to $\frac{1}{4}$ since all of the capillaries are of the same radius and therefore possess the same hydraulic conductivity. Under these conditions, the flow conservation equation is equivalent to the Laplace equation for the pressure, i.e. $\nabla^2 P=0$.

According to the invasion percolation model, the interface advances through the channels which provide the lowest capillary pressure which opposes the displacement of a wetting fluid by a non-wetting fluid (drainage).

An algorithm has been developed [4-6] based upon the above stochastic models and on the notion of the phase diagram in order to predict the transitions from one domain to another on the phase diagram. According to this algorithm, the interfacial tension is taken into account whenever the capillary number is less than the capillary number corresponding to the DLA or stable displacement boundaries. Motion of the interface occurs according to the DLA, anti-DLA or invasion percolation mechanisms with a phase transition probability [6] expressed in terms of the capillary number and the capillary numbers at the boundaries of the phase diagram. Therefore, at a low viscosity ratio, in the DLA domain, the interface moves according to the DLA model and with the random walkers being allowed to wander with a transition probability, p, at each step. At a capillary number between the DLA and the invasion percolation boundaries, motion of the interface occurs by both the DLA and the invasion percolation models. The decision of which model is chosen at each step is made by tossing a coin with a probability equal to the phase transition probability [6]. The role of the interfacial tension is taken into account by the invasion percolation mechanism. Finally, at a low capillary number, beyond the invasion percolation limit, advancement of the interface occurs by the invasion percolation mechanism. The transition from stable displacement to capillary fingering at high viscosity ratios is described in a similar way according to the anti-DLA and invasion percolation models.

According to the present model, (i) only one pore is invaded by the displacing non-wetting fluid at a time, (ii) both fluids are incompressible, (iii) trapping of the displaced fluid is allowed to occur, (iv) the interfacial tension is taken into account whenever the capillary number is less than the capillary number corresponding to the DLA or stable displacement boundaries, and (v) local anisotropy and heterogeneity resulting from the different sizes of the channels in the network are taken into account by the transition probability of the random walker at each step.

The actual simulations were performed on a square network of size 100×100 and for the physical parameters corresponding to the experiments of Lenormand *et al* [12]. The non-wetting fluid is injected on one side of the network (injection side) to displace the wetting fluid from the opposite side (recovery side). A trapping algorithm was incorporated in order to prevent invasion of trapped areas of the displaced fluid by the displacing fluid. The simulation stops when the interface first reaches the recovery side.

3. Results and conclusions

Typical simulations are shown in figures 1 and 2 for two particular viscosity ratios and for the range of capillary numbers presented in table 1. When the viscosity of the displacing fluid is less than that of the displaced fluid ($M = 2 \times 10^{-5}$), then at high capillary numbers (figure 1(a, b)) the fingers grow towards the exit and only a few

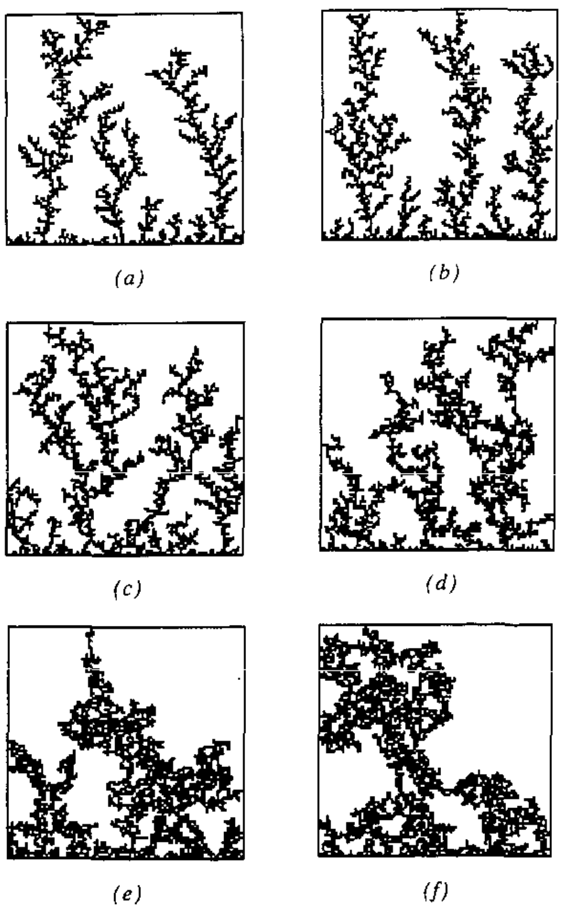

Figure 1. Numerical experiments for $M = 2.0 \times 10^{-5}$ and different capillary numbers: (a) $Ca = 5.0 \times 10^{-6}$, (b) $Ca = 5.0 \times 10^{-7}$, (c) $Ca = 1.0 \times 10^{-7}$, (d) $Ca = 1.0 \times 10^{-8}$, (e) $Ca = 5.0 \times 10^{-9}$, (f) $Ca = 1.0 \times 10^{-9}$.

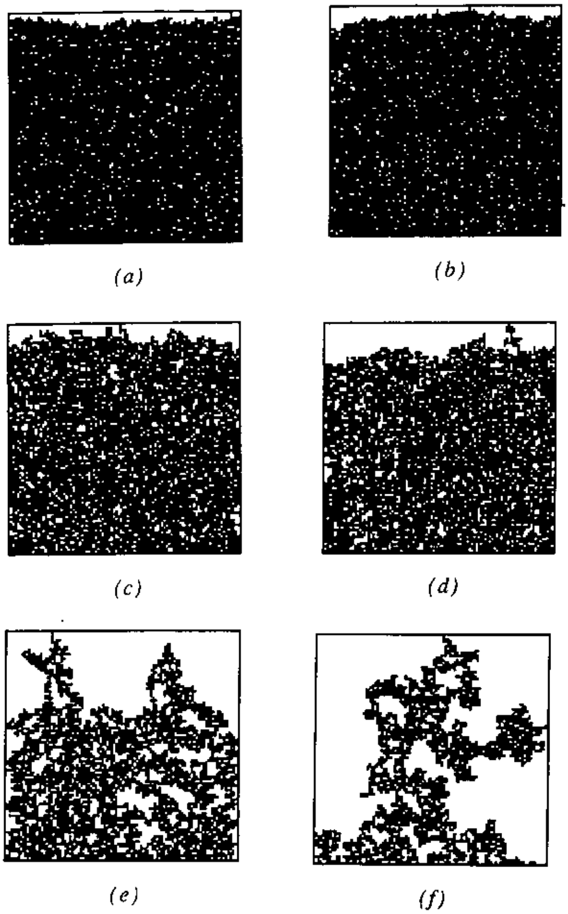

Figure 2. Numerical experiments for $M = 5.0$ and different capillary numbers: (*a*) $Ca = 3.4\times10^{-1}$, (*b*) $Ca = 3.4\times10^{-3}$, (*c*) $Ca = 3.4\times10^{-5}$, (*d*) $Ca = 1.4\times10^{-5}$, (*e*) $Ca = 2.7\times10^{-6}$, (*f*) $Ca = 2.3\times10^{-7}$.

small islands of displaced fluid are formed. At low capillary numbers, capillary forces become significant and the fingers grow all over resulting in more and larger islands of trapped displaced fluid as the capillary limit is reached (figure 1(*f*)). On the other hand, at a high viscosity ratio ($M = 5$) and at high capillary numbers the front is effectively flat and the instabilities are of the pore scale, resulting in a large number of islands of small size (figure 2(*a*, *b*)). At lower capillary numbers (figure 2(*c*–*f*)) the instabilities grow, resulting in fingers and islands of different sizes. The results presented in table 1 in all cases represent the average of 4–6 simulations.

The number of islands of size s at the end of each run is represented by $n(s)$. The size of an island is characterized in terms of a number of pores (nodes). For evaluation purposes, the islands are grouped into the specific size ranges 1, 2–3, 4–7, 8–15, . . . , [1], and the number of islands within a particular size range is given by

$$m(q) = \sum_{s=2^{q-1}}^{s=2^q-1} n(s) \qquad q = 1, 2, 3, 4, \ldots \tag{5}$$

where q characterizes the size range.

Table 1. Results of the simulations.

Figure	*Ca*	*M*	α
1(*a*)	5.0×10^{-6}	2.0×10^{-5}	2.28
1(*b*)	5.0×10^{-7}	2.0×10^{-5}	2.26
1(*c*)	1.0×10^{-7}	2.0×10^{-5}	2.13
1(*d*)	1.0×10^{-8}	2.0×10^{-5}	2.00
1(*e*)	5.0×10^{-9}	2.0×10^{-5}	1.97
1(*f*)	1.0×10^{-9}	2.0×10^{-5}	1.95
2(*a*)	3.4×10^{-1}	5.0	5.20
2(*b*)	3.4×10^{-3}	5.0	4.45
2(*c*)	3.4×10^{-5}	5.0	2.77
2(*d*)	1.4×10^{-5}	5.0	2.55
2(*e*)	2.7×10^{-6}	5.0	2.00
2(*f*)	2.3×10^{-7}	5.0	1.95

Following Sherwood's approach [1], the island size distribution is assumed to satisfy the relation

$$n(s)\propto s^{-\alpha} \tag{6}$$

where α is a function of M and Ca.

Upon combining equations (5) and (6), by replacing sums by integrals, and then assuming a proportionality constant β, we obtain

$$\begin{aligned} m(q) &= \beta\frac{2^{(q-1)(1-\alpha)}}{(\alpha-1)}(1-[2(1-2^{-q})]^{1-\alpha}) \\ &\simeq \beta\frac{2^{(q-1)(1-\alpha)}}{(\alpha-1)}(1-2^{1-\alpha}) \qquad \text{when } 2^{-q}\ll 1. \end{aligned} \tag{7}$$

In figures 3(a–f) and 4(a–f) we plot $\log_2[m(q)]$ against q (the size range) for the data shown in figures 1(a–f) and 2(a–f), respectively. Only islands surrounded completely by the displacing phase are taken into account. The slopes of the best-fitting straight lines in figures 3 and 4 correspond to the values of $1-\alpha$ in equation (7). The values of α determined from these plots are presented in table 1 and they represent the average of 4–6 simulations for each case. It may be observed that at a low viscosity ratio and high capillary numbers (figure 1(a–c)), the values of α are quite close to that predicted by Sherwood [1], namely 2.07. As the capillary number decreases α tends to an approximate value of 1.95, which corresponds to the capillary limit. For a high viscosity ratio, α decreases from 5.2 at a high capillary number (figure 2(a)) and tends to a value of 1.95 at low capillary numbers (figure 2(f)). The high values of α in the stable displacement domain (figure 2(a, b)) are to be expected since the vast majority of the islands here are of the pore size and only a few islands are of larger size. When decreasing the capillary number more islands of large size are formed and thus the value of α decreases. However, the fitting of the straight lines in figure 4(a, b) is not entirely satisfactory, suggesting that a power law behaviour of the island size distribution may be questionable in these two cases.

To better understand some of the consequences of the above distributions it is convenient to assume that the number of islands of size s is equal to $\beta s^{-\alpha}$ (equation (6)) and that the largest island is of size $O(N^{0.5})$, where N is the total number of pores

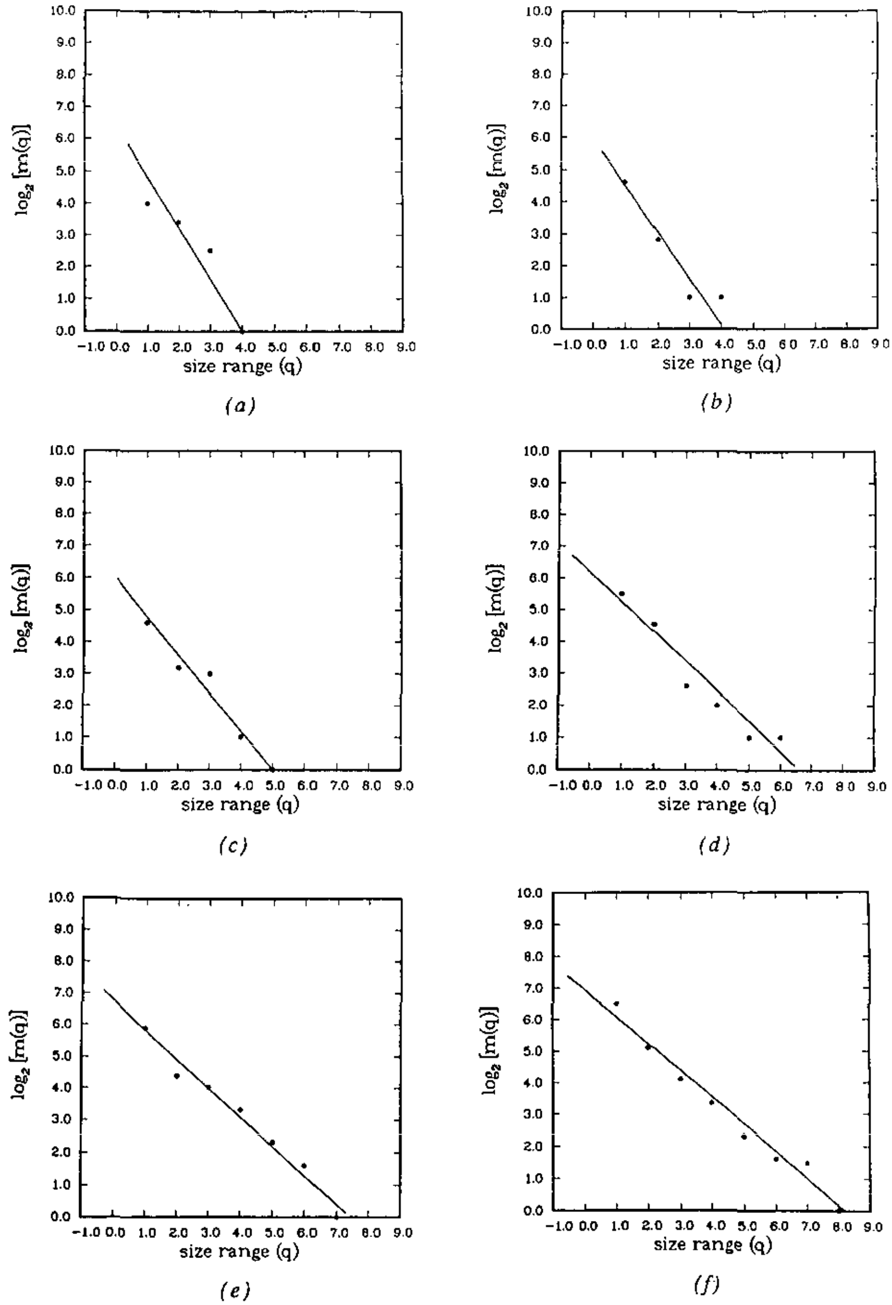

Figure 3. $\log_2[m(q)]$ against q for $M = 2.0\times10^{-5}$ and different capillary numbers: (*a*) $Ca = 5.0\times10^{-6}$, (*b*) $Ca = 5.0\times10^{-7}$, (*c*) $Ca = 1.0\times10^{-7}$, (*d*) $Ca = 1.0\times10^{-8}$, (*e*) $Ca = 5.0\times10^{-9}$, (*f*) $Ca = 1.0\times10^{-9}$.

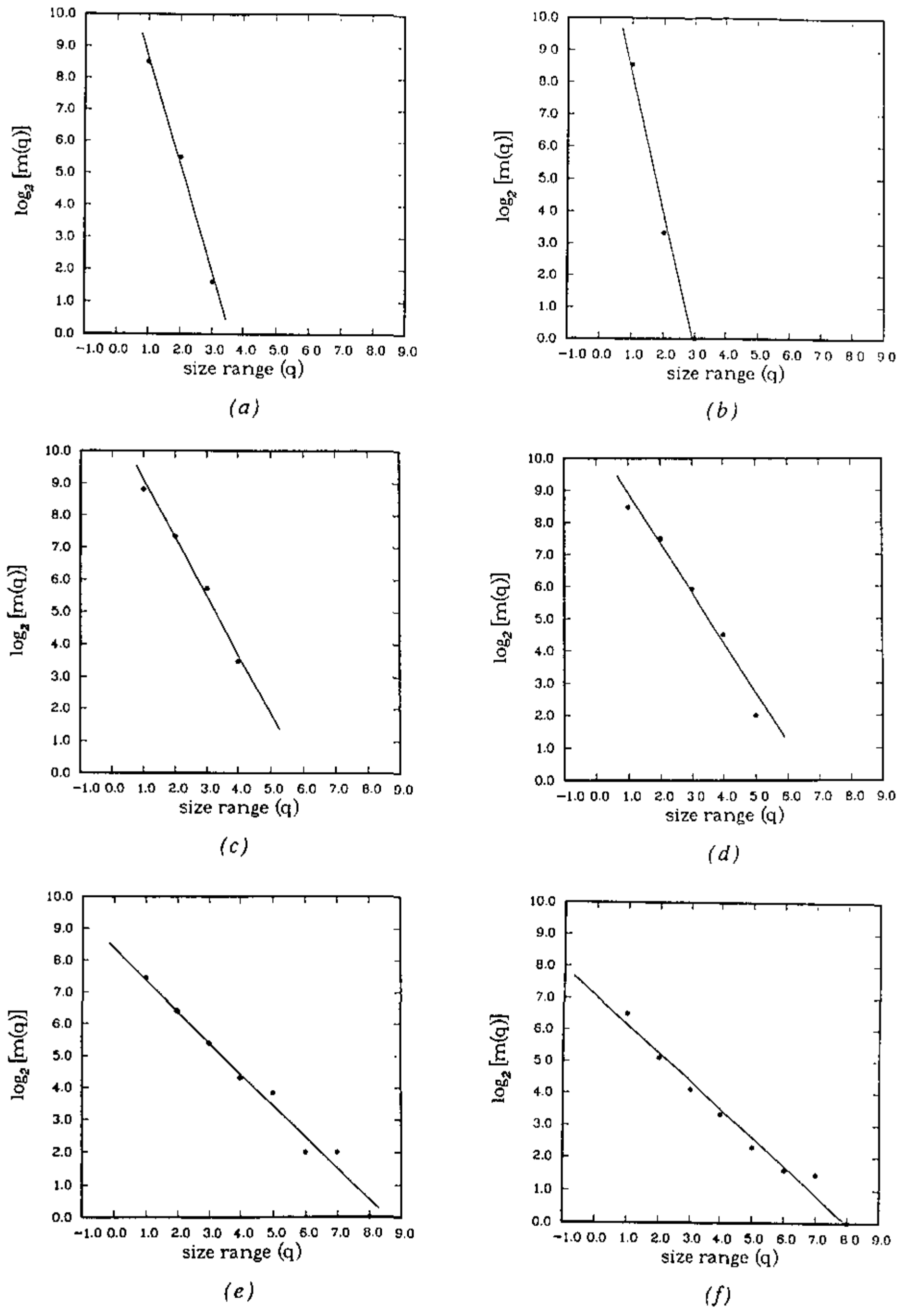

Figure 4. $\log_2[m(q)]$ against q for $M = 5.0$ and different capillary numbers: (*a*) $Ca = 3.4\times10^{-1}$, (*b*) $Ca = 3.4\times10^{-3}$, (*c*) $Ca = 3.4\times10^{-5}$, (*d*) $Ca = 1.4\times10^{-5}$, (*e*) $Ca = 2.7\times10^{-6}$, (*f*) $Ca = 2.3\times10^{-7}$.

in the network. Then, the total area of the islands will be

$$A=\beta \sum_{s=1}^{s=N^{0.5}} s^{-\alpha}s \sim \frac{\beta}{2-\alpha}(N^{1-\alpha/2}-1) \tag{8}$$

and the areal density of the islands in the entire network will be

$$f=\frac{A}{N}=\frac{\beta}{2-\alpha}(N^{-\alpha/2}-N^{-1}). \tag{9}$$

According to the last relation, the density of the island area with respect to the size of the network decreases rapidly at high viscosity ratios for large values of α (stable displacement domain). However, the density decreases more gradually at lower capillary numbers (transition domain) and increases slowly in the capillary domain. At low viscosity ratios the density increases slowly in the DLA and the transition domains and decreases slowly as soon as the capillary limit is reached.

In conclusion, a study based on a previous approach [1] has been carried out in order to examine the effects of capillary forces on the island size distribution for immiscible displacement flow in porous media. From the above results it appears that a power law behaviour of the island size distribution exists, at least at low viscosity ratios, with a value of α close to 2.0. However, the values of α determined from figure 4(a, b) are in question. It is not clear if this represents a limitation of a power law behaviour in this domain. The authors believe that further studies on a larger scale are required in order to elucidate these matters.

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